MAXIMIZING VERSUS MATCHING ON CONCURRENT VARIABLE-INTERVAL SCHEDULES

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Maximization and matching predictions were examined for a time-based analogue of the concurrent variable-interval variable-ratio schedule. One alternative was a variable interval whose time base operated relatively independent of the schedule chosen, and the other was a discontinuous variable interval for which timing progressed only when selected. Pigeons switched between schedules by pecking a changeover key. The maximization hypothesis predicts that subjects will show a bias toward the discontinuous variable interval and undermatching; however the obtained results conformed closely to the predictions of the matching law. Finally, a quantitative comparison was made of the bias and sensitivity estimates obtained in published concurrent variable-interval variable-ratio analogue studies. Results indicated that only the ratio-based analogue of the concurrent variable interval variable ratio studied by Green, Rachlin, and Hanson (1983) produced significant bias toward the variable-ratio alternative and undermatching, as predicted by reinforcement maximization.

Key words: choice, maximization, optimization, matching, melioration, concurrent interval schedules, reinforcement, key peck, pigeons

Some schedules used in operant choice experiments progress toward reinforcement only when chosen; other schedules operate whether or not they have been chosen. For example, a variable-ratio (VR) schedule makes reinforcement contingent upon a certain number of responses, and the schedule advances only when it is selected and the subject responds. In contrast, the timer of a variable-interval (VI) schedule runs almost continuously, whether selected or not. The continuous timing of the VI (as typically programmed) is not absolute, however, because once a reinforcer is set up and is waiting to be collected, the schedule stops, reducing the overall reinforcement density. Linear VI schedules, on the other hand, approximate completely continuous timing (see Vaughan, 1986; Vaughan & Miller, 1984).

When concurrent operants combine a schedule that can advance only when chosen with one that can advance whether chosen or not, theories of choice based on the principle of reinforcement maximization predict respond-

ing that deviates from matching. In the case of the concurrent VI VR, for example, reinforcement is maximized by a bias toward the VR schedule and undermatching. By spending the majority of time responding on the VR and making brief, periodic visits to the VI, the subject keeps the VR schedule advancing and the interval timer running. Contrary to this prediction, however, Herrnstein and Heyman (1979) found that pigeons' responding on concurrent VI VR closely conformed to the matching law and failed to maximize overall reinforcement.

Supporters of maximizing theory have suggested a number of explanations for this discrepancy. Green, Rachlin, and Hanson (1983) argued that the effect of relative leisure on the VI schedule was responsible for the lack of bias toward the VR. They pointed out that VI schedules provide the opportunity to engage in other behavior while the interval timer progresses, whereas VR schedules are "labor intensive." As a result, Green et al. (1983) suggested that the greater food reinforcement that would have resulted from bias toward the VR had been counterbalanced by the value of relative leisure on the VI. Alternatively, Silberberg and his colleagues (Sakagami, Hursh, Christensen, & Silberberg, 1989; Shurtleff & Silberberg, 1990) hypothesized that subjects maximize their daily food intake ("income level") rather than session intake ("income

flow"). In the Herrnstein and Heyman (1979) experiment, pigeons were fed in their home cages to maintain them at 80% of their free-feeding weights, making income level independent of income flow. According to this view, the impossibility of maximizing income level under these conditions allowed "alternative processes to govern choice" (Shurtleff & Silberberg, 1990, p. 274). Finally, Rachlin, Green, and Tormey (1988) proposed an explanation based on the reduced value of VI reinforcers that set up while birds responded on the VR. According to this account, "extra" reinforcers are *earned* when they set up on the VI while the subject responds on the VR; however, these extra reinforcers are *obtained* only after returning to the VI. As a result, the predicted bias toward the VR schedule falls victim to extra reinforcers that are weakened by delays between earning and obtaining. Tests of these accounts have, in some cases, produced ambiguous results that fall between those predicted by maximization theory and the matching law; other tests have produced results comparable to Herrnstein and Heyman (1979).

The goals of the present study were twofold. First, we wished to test the maximizing prediction in the context of concurrent VI discontinuous VI schedules. The discontinuous VI is identical to a standard VI, except that timing stops when the subject switches to the other schedule. Both DeCarlo (1985) and Heyman and Herrnstein (1986) addressed the leisure hypothesis by using response-independent concurrent variable-time (VT) variable-ratio-time (VRT) schedules. The VT programmed reinforcement in a fashion analogous to a response-dependent VI. Reinforcers that set up on the VT while the VRT schedule was chosen were delivered upon return to the VT. In contrast, on the VRT time progressed and reinforcers set up only when the schedule had been chosen. The two schedules were associated with keylights or houselights of different colors, and subjects alternated schedules by pecking a changeover key. The present study examined response-dependent versions of these schedules.

A second goal was to make a quantitative comparison of the bias and sensitivity parameters obtained in this study and in previous investigations of concurrent VI VR analogue schedules (DeCarlo, 1985; Green et al., 1983; Heyman & Herrnstein, 1986). In each of these cases, the general type of schedule (ratio or interval) was balanced across alternatives and either response matching, time matching, or both, was reported.

METHOD

Subjects

Four White Carneau pigeons served as subjects. The pigeons, all of which had previous experience in operant conditioning experiments, were tested at between 75% and 85% of their caged free-feeding weights. As necessary to maintain body weight, additional mixed grain was given in the home cage following each session.

Apparatus

The experiment was conducted in a standard experimental chamber. The response panel was 37 cm wide and 33 cm from the back wall of the chamber. The panel contained three response keys mounted 23 cm above a wire mesh floor and separated by 10 cm, center to center, but only the left and center keys were used in the present study. The response keys required a force of 0.15 N to operate, and operation of a key produced a brief feedback click. Each key could be transilluminated from the rear with different colored lights. Access to the food hopper was through a recessed opening 13 cm below the center key and 10 cm above the floor. The chamber was illuminated by a white houselight mounted in a rectangular aluminum frame with a translucent cover (12 cm by 11 cm) suspended in the rear right corner of the chamber. A fan provided masking noise. The experiment was controlled by a PDP® 8a computer located in an adjacent room.

Procedure

A changeover-key concurrent procedure (Findley, 1958) was used to schedule reinforcement. Responses on the center (main) key were occasionally reinforced with grain. A single response on the left (changeover) key alternated the key color and schedule on the center key. In the first four conditions, when the center key was red, the discontinuous VI schedule was in effect. A response to the changeover key halted the operation of the discontinuous VI schedule, changed the color of

Table 1

Average number of responses on a schedule, average time at a schedule, average number of reinforcers received from a schedule, and average changeovers per session over the last nine sessions of each condition.

Subject	Schedule VI	Dis VI[a]	Sessions	Responses VI	Dis VI	Time (min) VI	Dis VI	Reinforcers VI	Dis VI	Changeovers
491	30	30	15	1,520.2	92.3	27.4	1.8	56.4	3.6	42.4
	60	30	19	765.3	1,268.3	7.5	19.1	22.8	37.2	203.0
	45	30	15	674.2	1,082.7	7.3	16.9	26.6	33.4	183.7
	45	15	15	176.9	852.1	1.7	12.4	12.2	47.6	56.9
	45	45[b]	15	2,507.8	177.0	38.5	2.8	53.8	3.4	99.3
	60	45[b]	65	1,532.9	1,489.7	12.9	21.6	31.2	28.8	284.1
	90	45[b]	19	1,359.1	1,894.4	11.4	27.2	23.8	35.9	313.1
451	30	30	15	957.4	413.3	17.2	7.5	45.6	14.4	398.7
	60	30	15	763.7	1,020.6	13.6	16.5	27.3	32.7	413.8
	45	30	19	474.2	864.4	10.6	15.1	30.4	29.6	344.7
	45	15	25	145.1	714.4	4.3	11.2	16.8	43.4	156.2
	45	45[b]	16	2,196.9	208.7	36.2	4.3	52.6	5.6	243.7
	60	45[b]	27	1,859.8	1,065.9	27.3	13.5	39.0	17.7	400.9
	90	45[b]	15	780.8	1,694.6	12.6	27.3	24.0	36.0	316.7
429	30	30	27	867.0	489.6	13.7	9.7	41.1	18.9	169.8
	60	30	15	656.3	1,009.7	10.6	18.2	25.0	35.0	160.9
	45	30	16	770.1	1,025.3	13.8	13.7	32.8	27.2	172.0
	45	15	15	313.3	691.5	5.5	10.9	18.3	41.7	90.4
	45	45[b]	21	2,047.7	457.0	28.7	9.1	48.1	11.8	211.1
	60	45[b]	28	1,554.8	1,169.9	20.1	18.2	35.7	24.0	248.1
	90	45[b]	15	1,250.0	1,801.2	15.1	24.9	25.0	32.8	184.1
427	30	30	15	1,826.9	2.9	30.5	0.1	59.8	0.2	4.4
	60	30	15	699.5	1,167.8	11.3	18.1	24.3	35.7	188.6
	45	30	21	857.4	995.2	11.7	14.8	30.8	29.2	200.1
	45	15	25	175.0	835.1	2.7	12.2	13.0	47.0	76.5
	45	45[b]	20	1,712.1	593.3	27.5	9.9	46.8	12.6	218.4
	60	45[b]	22	1,525.9	1,032.5	21.9	17.5	36.3	23.1	354.2
	90	45[b]	15	1,468.2	1,768.2	17.3	24.0	26.1	31.7	440.9

[a] Discontinuous VI.

[b] In these conditions, the discontinuous VI key color was green and VI red, the reverse of previous conditions.

the center key to green, and provided access to reinforcement from the VI schedule. This VI timed continuously when the key was green or red, but dispensed reinforcers only when the light was green.

The changeover key was always illuminated white, and changeovers to the VI produced a 2.0-s changeover delay (COD) before a response could be reinforced. A COD was not programmed on the discontinuous VI, because reinforcers could only set up while the schedule was in effect. To assess color preferences, the key colors associated with the standard and discontinuous VI schedules were reversed for the last three conditions of the experiment. The order of conditions was the same for each bird and is listed on the left side of Table 1. Schedule values were chosen to provide a range of choice proportions. Experience with equal

schedule values in the first condition produced preferences for the continuous VI ranging from 2:1 to 630:1; therefore, schedule values either favored the discontinuous VI or were equal. Sessions were conducted daily and terminated after 60 reinforcers or 45 min, whichever came first. Conditions were changed when responding had become stable according to the following criteria: On the 15th day and each day thereafter, the last nine sessions were divided into three blocks of three consecutive sessions. The means of the proportion of time spent on the VI were calculated for each session block, and responding was considered stable if the means of the three blocks differed by no more than 0.05 and showed no increasing or decreasing trend (i.e., $M_1 < M_2 < M_3$ or $M_1 > M_2 > M_3$). The results presented are averages over the last nine sessions of each condition.

RESULTS

An Assessment of Matching

The results are presented in Table 1. Bias and sensitivity parameters were estimated using the method of least squares applied to Baum's (1974) log(base 10)-ratio form of the generalized matching law:

$$\log(B_1/B_2) = a \log(R_1/R_2) + \log b,$$

where B_1 is time or responses on the VI schedule, B_2 is time or responses on the discontinuous VI schedule, R_1 and R_2 are the respective obtained reinforcers, and a and b are the fitted sensitivity and bias parameters, respectively.

Figure 1 shows the relationship between the logarithm of the response ratios and the logarithm of the reinforcement ratios for each subject and for the group as a whole. The obtained slopes for Birds 429 and 491 closely approximated 1.0, whereas the slopes for Birds 451 and 427 showed slight overmatching. The intercepts for all 4 birds were negative and ranged from −0.03 to −0.17. These negative intercepts indicate that all birds showed some degree of bias, responding more on the discontinuous VI schedule than would be predicted from the obtained reinforcement rates on each schedule. Estimates of the percentage of variance accounted for ranged from 94% to 99%. The results for the group data were similar to those obtained for individual subjects. In this case, the slope was 1.12 and the intercept was −0.08; the fitted line accounted for over 98% of the variance.

Figure 2 shows the assessment of matching with respect to time allocation on each schedule. Slopes for Birds 451 and 427 closely approximated 1.0, whereas Bird 491 showed slight overmatching and Bird 429 showed undermatching. Again, all the intercepts were negative, indicating a slight, though consistent, bias towards the discontinuous VI. Estimates of variance accounted for ranged from 96% to 99%.

In summary, although individual subjects showed minor deviations from normative matching, in general the results of this experiment conformed closely to the predictions of the matching law. The r^2 values for all 10 fitted functions were greater than .940, indicating that the variance of both time and response allocation could be accounted for by the obtained reinforcement ratios.

An Assessment of Reinforcement Maximization

To evaluate how closely the data approximated overall reinforcement maximization, the time proportions on the VI that would have maximized overall reinforcement in each condition were calculated using a modified version of the expected reinforcement model used by Heyman and Herrnstein (1986; see Appendix). As a first step, the adequacy of the model was evaluated by substituting the obtained time ratios and comparing the predicted overall rates of reinforcement to those actually obtained. Figure 3 presents a plot of the predicted reinforcement rates as a function of obtained reinforcement rates. A least squares regression line fitted to these points closely approximated a slope of 1.0 and accounted for 99% of the variance. Although this analysis suggests that the model accurately predicted the obtained results, the intercept of −7.9 indicates that the model underestimated actual rates of reinforcement by approximately eight reinforcers per hour. Previous studies (Herrnstein & Heyman, 1979; Heyman & Herrnstein, 1986) found the model to predict more accurately the obtained rates of reinforcement, but in each case the COD term was not used in the calculation of predicted rates of reinforcement. Although the full model underestimated the absolute rates of reinforcement obtained, the r value of .99 indicates that relative rates of reinforcement are predicted very accurately.

To determine the allocation ratios that would have maximized overall reinforcement, the proportion of time on each schedule entered into the expected reinforcement model was varied for each subject, in an iterative manner, until a maximum expected reinforcement value was obtained. The results of this analysis are presented in Figure 4. The filled symbols replot the observed time and reinforcement ratios from the group data in Figure 2, and the open symbols plot the time and reinforcement ratios that would have maximized overall rates of reinforcement. A least squares regression line fitted to the points predicted by reinforcement maximization shows relatively weak correlation between time and reinforcement ratios, $r(26) = .655$, substantial bias toward the discontinuous VI, $b = −0.81$, and undermatching, $a = 0.57$. Finally, according to these calculations, by responding as they did, the subjects lost an average of 23.3 reinforcers per hour (range, 18.8 to 28.0).

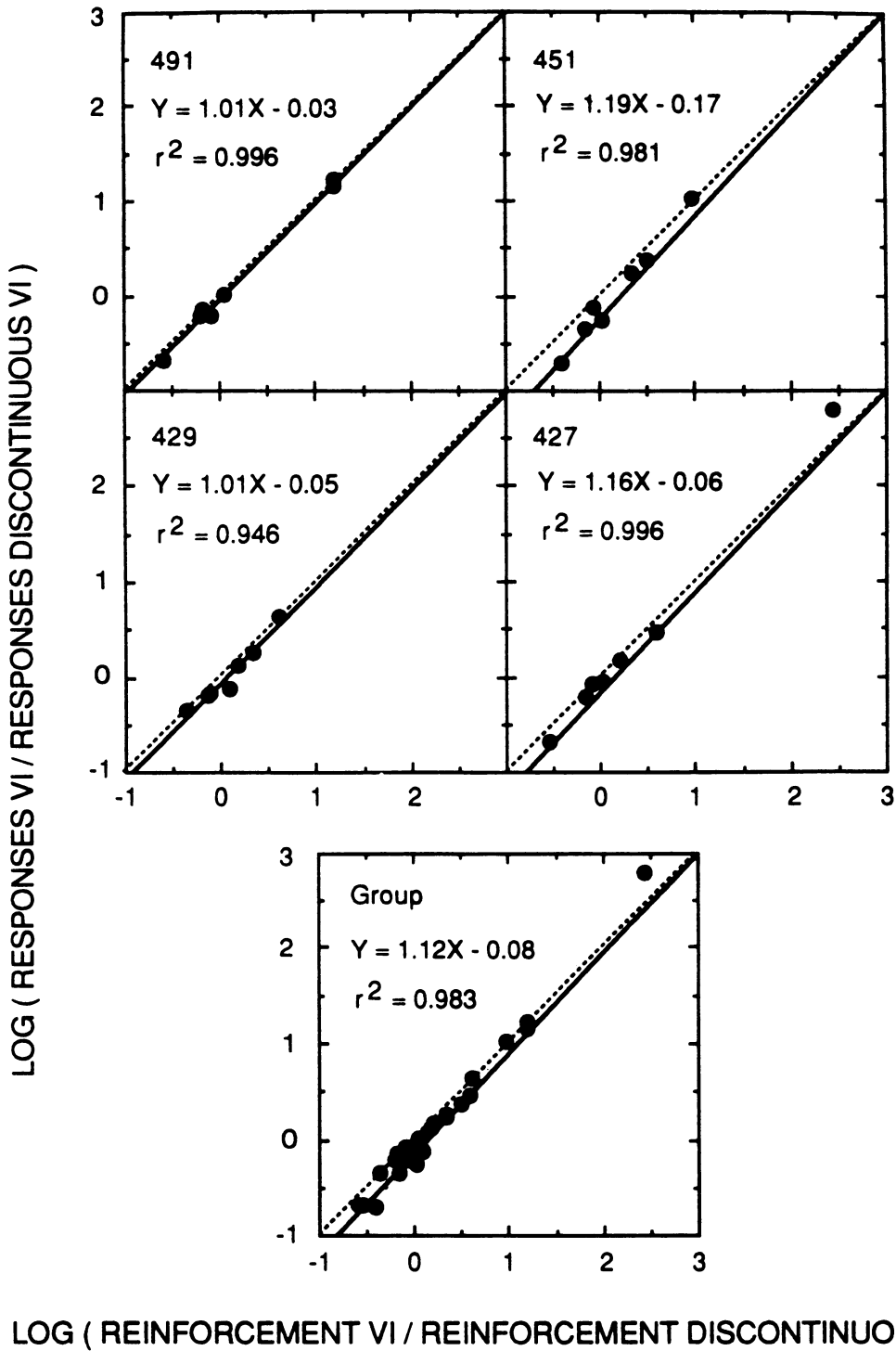**LOG (REINFORCEMENT VI / REINFORCEMENT DISCONTINUOUS VI)**

Fig. 1. Log ratio of responses on the VI over responses on the discontinuous VI as a function of the log ratio of reinforcers on the VI over reinforcers on the discontinuous VI. Results for each bird and the group are shown. Broken lines represent normative matching, and solid lines are fitted least squares regression lines. Bias, sensitivity, and estimates of variance accounted for are provided.
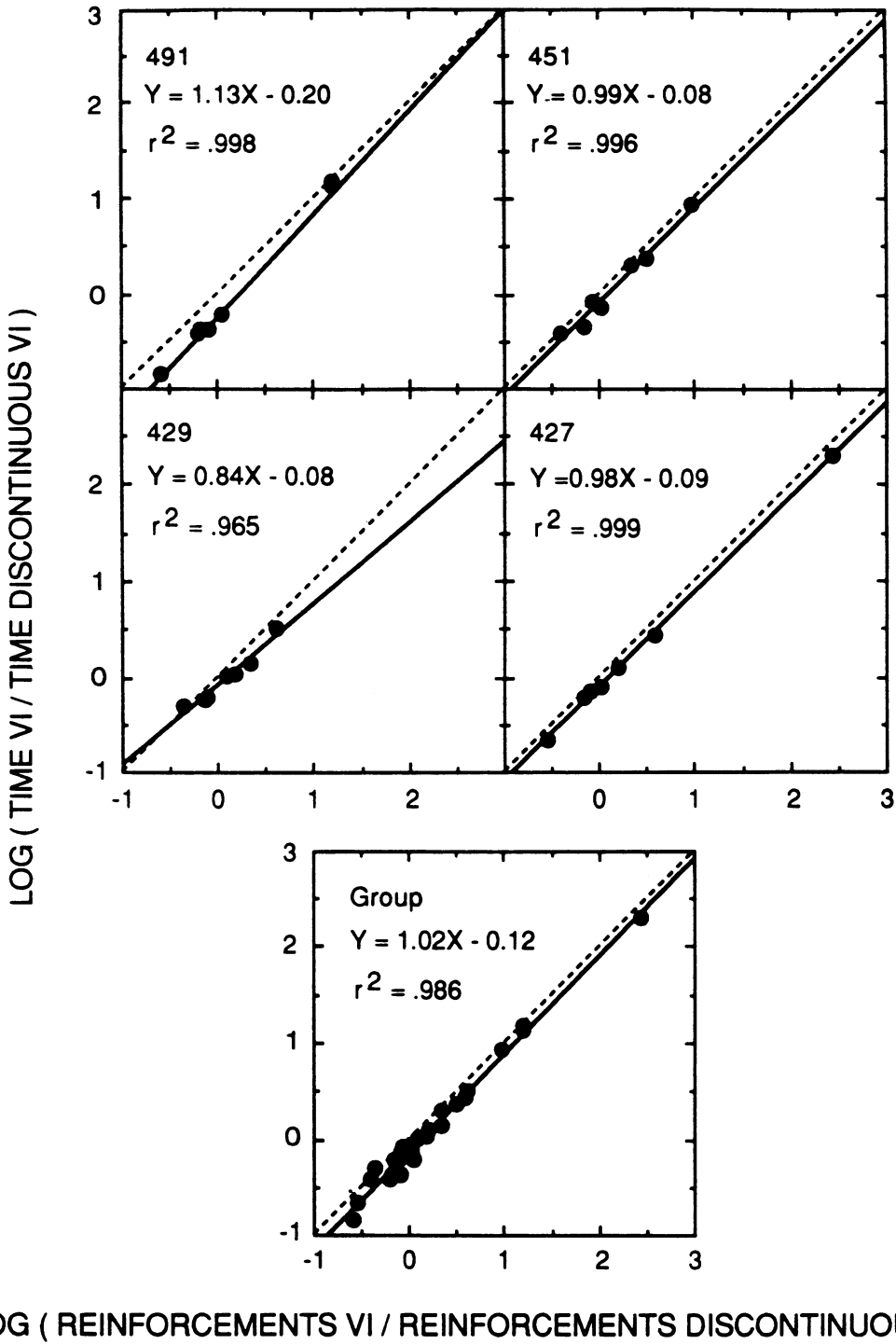
Fig. 2.   Log ratio of time on the VI over time on the discontinuous VI as a function of the log ratio of reinforcers on the VI over reinforcers on the discontinuous VI. Results for each bird and the group are shown. Broken lines represent normative matching, and the solid lines are fitted least squares regression lines. Bias, sensitivity, and estimates of variance accounted for are provided.
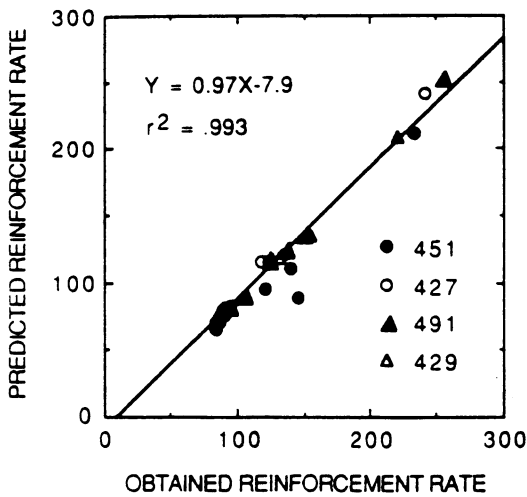
Fig. 3. Reinforcers per hour as predicted by the model presented in the appendix as a function of the actual obtained rate of reinforcement. A different symbol is used for each bird. The least squares regression line is shown, as are the regression coefficients and variance accounted for.
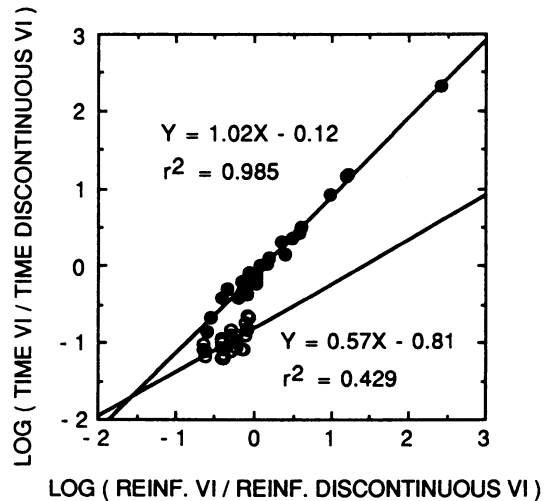

Fig. 4. The obtained group log time and reinforcement ratios (filled circles) replotted from Figure 2, and the log time and reinforcement ratios that would have maximized overall rates of reinforcement (open circles). The least squares regression lines are shown for each set of points, as are the estimates of bias, sensitivity, and variance accounted for.

DISCUSSION

A Quantitative Review of Past and Current Findings

The present investigation is one of a series of experiments that have extended the findings of Herrnstein and Heyman's (1979) study of concurrent VR VI schedules by examining concurrent operants that have matched response and reinforcement requirements yet retain the essential feature of contrasting effects of choice on the operation of each schedule. These analogue experiments have included response-independent time-based schedules (DeCarlo, 1985; Heyman & Herrnstein, 1986; Rachlin et al., 1988), response-based schedules (Green et al., 1983), and response-dependent time-based schedules (Shurtleff & Silberberg, 1990, Experiment 3; the present experiment). In each case, reinforcement maximization predicted a bias toward the schedule that operated only when selected.

To compare the present results to those of previous investigations, analyses of variance (ANOVAs) were conducted on the bias and sensitivity values obtained. A study was included in these analyses if it (a) used a concurrent VI VR or a time- or response-based analogue to the concurrent VI VR and (b) reported bias and sensitivity values for either time or responses. Although both one- and two-

key procedures were used, all of the studies employed pigeons pecking keys for access to grain. The time and response data included in these analyses are presented in Tables 2 and 3. Single-factor ANOVAs conducted on the bias and sensitivity terms in Table 2 revealed no significant differences across the four studies, $F(3, 13) = 1.38$, and $F(3, 13) = 1.05$, respectively.

Single-factor ANOVAs conducted on the studies that assessed response matching (see Table 3) revealed statistically significant differences in both bias, $F(2, 9) = 26.46$, $p < .001$, $\omega^2 = .81$, and sensitivity, $F(2, 9) = 5.32$, $p < .05$, $\omega^2 = .44$. Tukey HSD post hoc tests showed that the Green et al. (1983) study produced significantly higher mean bias than the other studies and significantly lower sensitivity than the present study ($p < .05$). No other comparisons yielded statistical significance.

The results of these analyses are remarkably clear. Only the Green et al. (1983) ratio analogue of the concurrent VI VR experiment produced significant bias toward the VR and undermatching as predicted by global maximizing. Their procedure involved a modified concurrent VR VR schedule for which pecks on either key advanced the schedule associated with the right key, whereas only pecks on the

Table 2

Bias and sensitivity values for time allocation on concurrent VI discontinuous VI schedules.

Subject	Herrnstein and Heyman (1979)	DeCarlo (1985)	Heyman and Herrnstein (1986)	Present study
Log(bias)[a]				
1	−0.10	−0.16	−0.05	−0.20
2	−0.22	0.04	−0.08	−0.08
3	−0.01	−0.09	0.01	−0.08
4	−0.47	−0.04	−0.05	−0.09
5				−0.14
M	−0.20	−0.06	−0.06	−0.11
SD	0.20	0.09	0.05	0.06
Sensitivity				
1	1.02	0.75	0.89	1.13
2	0.88	0.81	0.84	0.99
3	0.71	1.00	0.83	0.84
4	0.83	0.91	0.97	0.98
5				1.05
M	0.86	0.87	0.92	0.98
SD	0.12	0.11	0.09	0.12

[a] Expressed as the degree of bias toward the VI schedule.

Table 3

Bias and sensitivity values for response allocation on concurrent VI discontinuous VI schedules.

Subject	Herrnstein and Heyman (1979)	Green, Rachlin, and Hanson (1983)	Present study
Log(bias)[a]			
1	−0.08	−0.28	−0.03
2	−0.10	−0.43	−0.17
3	−0.19	−0.41	−0.05
4	−0.11	−0.49	−0.06
M	−0.12	−0.40	−0.08
SD	0.05	0.09	0.06
Sensitivity			
1	0.97	0.72	1.01
2	0.98	0.26	1.19
3	1.00	0.93	1.01
4	1.03	0.85	1.16
M	1.00	0.69	1.09
SD	0.03	0.30	0.10

[a] Expressed as the degree of bias toward the VI schedule.

left key advanced the schedule on that side. The obtained response distributions showed bias toward the standard VR that fell between the predictions of global maximizing and the matching law (see Table 3).

Rachlin et al. (1988) conducted the only other study to produce results in the direction of a maximizing prediction. Their procedure involved a concurrent VT VT schedule in which reinforcers were earned but not obtained on one side (analogous to the "stored" reinforcers of a standard VI schedule) and obtained but not earned on the other. The earning and obtaining of reinforcers were controlled by independent VT timers, and pigeons pecked a changeover key to switch between alternatives. Similar to the results of Green et al. (1983), obtained choice proportions fell between the predictions of matching and maximizing.

In their review of the study by Green et al. (1983), Heyman and Herrnstein (1986) suggested that subjects may have emitted response sequences that represented new response classes (Schwartz, 1980, 1981). They argued that these response classes, which could not be detected under the procedure used, may themselves have conformed to the predictions of the matching law. In the case of the study by Rachlin et al.

(1988), a similar hypothesis is justified. Although VT schedules were used, reinforcement could be obtained only by spending time on both components. It is likely that subjects developed alternating response sequences involving both measured (changeover) and other unmeasured responses whose larger units were reinforced like a single operant.

It could be argued that the present results were affected by the lack of a COD on the discontinuous VI schedule. Perhaps this asymmetry prevented the development of the expected bias. This procedure was chosen to duplicate the programming of Heyman and Herrnstein's (1986) response-independent versions of the schedules used in the present study. Although reinforcers could not set up on the discontinuous VI while the other schedule was selected, it is likely that some reinforcers were obtained within 2 s of a switch to the discontinuous VI. Nevertheless, were these reinforcers to affect choice, they could be expected to strengthen switching to the discontinuous VI, consistent with the maximizing hypothesis. All 4 pigeons showed a slight bias toward the discontinuous VI schedule, which is perhaps attributable the asymmetrical COD, but the results did not support maximizing.

Further evidence that the lack of a COD on the discontinuous VI schedule did not significantly affect the results of this study can be

found in Table 2. The Heyman and Herrn-stein (1986) experiment was a replication of the response-independent concurrent VI VR analogue examined by DeCarlo (1985). Schedules were programmed in exactly the same manner, with one exception: DeCarlo (1985) used symmetrical 2-s CODs, whereas Heyman and Herrnstein (1986) did not pro-gram a COD on the discontinuously timing VT. The bias and sensitivity estimates pre-sented in Table 2, as well as the ANOVA findings, show that, despite this discrepancy, the two studies produced very similar results.

Conclusion

The present study tested maximizing and matching accounts of choice by exposing pi-geons to concurrent VI discontinuous VI schedules. Similar to several previous studies (DeCarlo, 1985; Herrnstein & Heyman, 1979; Heyman & Herrnstein, 1986) and consistent with the predictions of the matching law, ob-served response and time distributions could be explained by matching with respect to the obtained reinforcement ratios. Subjects failed to maximize their earnings, losing reinforcers at an average rate of over 20 per hour.

The quantitative review of previous findings showed remarkable consistency for both re-sponse-dependent and response-independent interval schedules. This result is consistent with studies using complex time-based schedules (e.g., Vaughan, 1981) and with the view that local, not overall, rates of reinforcement con-trol choice (Herrnstein, 1982). Future re-search should be directed toward an improved understanding of the differences between the present results and those obtained by Green et al. (1983) and Rachlin et al. (1988).

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APPENDIX

The model used to calculate expected rates of reinforcement was a modification of Equa-tion A11 described in Heyman and Herrnstein (1986) and credited to Houston and McNa-mara (1981). It assumes (a) that interrein-forcement times are programmed exponen-tially (Fleshler & Hoffman, 1962) and (b) that changeovers from one schedule to the other occurred after a fixed time, a switching pattern that maximizes reinforcement. The version of

Equation A11 used in the present experiment was as follows:

$$E(R) = \frac{T_1 - C}{(T_1 + T_2)(VI + r_1)} + \frac{1 - e^{(-T_2 + C)/VI}}{(T_1 + T_2)}$$

$$+ \frac{T_2}{(T_1 + T_2)(disVI + r_2)},$$

where T_1 and T_2 are the proportions of time spent on the VI and discontinuous VI, respectively. C is the duration of the COD, and VI and disVI are the programmed values associated with each schedule. Finally, r_1 and r_2 are the average interresponse times on the VI and discontinuous VI schedules. The values for T_1, T_2, r_1, and r_2 for each subject were taken from Table 1.

Each of the quotients in this formula represents the three methods of obtaining reinforcement on the concurrent VI discontinuous VI schedule. The first quotient estimates reinforcers set up and received while working at the VI. The second quotient calculates the reinforcers set up on the VI while the subject is working at the discontinuous VI. These "stored" reinforcers are obtained only after returning to the VI. Finally, the last quotient represents reinforcers earned and obtained on the discontinuous VI schedule.